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The difference between n-dimensional regularization and n-dimensional reduction in QCD

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Abstract. We discuss the difference between n-dimensional regularization and n-dimensional reduction for processes in QCD which have an additional mass scale. Examples are heavy flavor production in hadron–hadron collisions or on-shell photon–hadron collisions where the scale is represented by the mass m. Another example is electroproduction of heavy flavors where we have two mass scales given by m and the virtuality of the photon $Q = \sqrt{-q^2}$. Finally we study the Drell–Yan process where the additional scale is represented by the virtuality $Q = \sqrt{q^2}$ of the vector boson (γ^*, W, Z). The difference between the two schemes is not accounted for by the usual oversubtractions. There are extra counter terms which multiply the mass scale dependent parts of the Born cross sections. In the case of the Drell–Yan process it turns out that the off-shell mass regularization agrees with n-dimensional regularization.

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Here we discuss the consistency between n-dimensional regularization and n-dimensional reduction for processes in quantum chromodynamics (QCD) which have an additional mass scale. This paper is a continuation of earlier work which dealt with jet physics in hadron–hadron collisions where no additional mass scale is present [1, 2]. The method of n-dimensional regularization was originally introduced in [3] with one exception: that the number of degrees of freedom of the gluon is now taken to be $n - 2$. All numerators of virtual and radiative graphs are represented in n dimensions. Likewise the loop integrals and phase space integrals are evaluated in n dimensions. For the gluon spin average one has the factor $1/(n - 2)$. The method of n-dimensional reduction was proposed in [4] (see also [5]). Apart from the number of external dimensions which is 4 instead of n (see Table 1) the numerators for virtual and radiative graphs are now presented for n equal to 4. However the loop integrals and phase space integrals are still evaluated in n dimensions. This implies that the tensorial reduction of the loop graphs and phase space integrals are still done in n dimensions. Only traces and the usual Lorentz algebra are done in four dimensions. The gluon spin average factor is now 1/2. We compare the two schemes in Table 1. If we perform both regularization techniques the usual divergences which appear in next-to-leading order (NLO) calculations are of the ultraviolet (UV), infrared (IR) and collinear (C) type and produce pole terms of the type $1/(n-4)^k$. After cancelling the IR and the final state C divergences by adding the results for the loop graphs to the squares of the radiative graphs we are left with the UV singularities and the initial state C divergences. This is true for inclusive processes only. Then we have to perform mass renormalization and coupling constant renormalization to get rid of the UV divergences. In this paper we choose the on-mass-shell scheme for mass renormalization in both regularization procedures, where

Table 1. Definitions of the numbers of degees of freedom in the two regularization prescriptions

	n-dim. regularization	n-dim. reduction
number of internal dimensions	n	n
number of external dimensions	n	4
number of internal gluons	$n-2$	2
number of external gluons	$n-2$	2
number of internal quarks	2	2
number of external quarks	2	2

$$\hat{m} = m \times \left[1 + C_F \frac{\alpha_s}{4\pi}\left(\frac{6}{n-4} + 3\gamma_E - 3\ln 4\pi - 4 - 3\ln\frac{\mu^2}{m^2}\right)\right]. \tag{1}$$

Here $\hat{m}$ and m denote the bare and renormalized mass respectively. Coupling constant renormalization is achieved in n-dimensional regularization in the $\overline{\mathrm{MS}}$ scheme by

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$$\hat{\alpha}_s = \alpha_s \left[1 + \frac{\alpha_s}{4\pi}\beta_0 \left\{\frac{2}{n-4} + \gamma_E - \ln 4\pi\right\}\right],$$
$$\beta_0 = \frac{11}{3} C_A - \frac{4}{3} T_f n_f, \quad (2)$$

where $\hat{\alpha}_s$ and α_s denote the bare and renormalized coupling constant respectively. The initial state C singularities are removed via mass factorization. The latter is achieved by subtracting the Born cross sections convoluted with kernels in which the residues of the pole terms are given by the splitting functions P_{ij} (for the normalization see (5.9) in [6]). Choosing the $\overline{\mathrm{MS}}$ scheme in n-dimensional regularization we have

$$\Gamma_{ij}(x) = \delta_{ij}\,\delta(1-x) + \frac{\alpha_s}{4\pi}\left[\frac{1}{2} P_{ij}(x)\left(\frac{2}{n-4} + \gamma_E - \ln 4\pi\right)\right]. \quad (3)$$

It is clear that both regularization procedures lead to finite results which are however different. These differences can be accounted for by performing an additional finite coupling constant renormalization and a finite mass factorization in the case of n-dimensional reduction. Here we use

$$\hat{\alpha}_s = \alpha_s \times \left[1 + \frac{\alpha_s}{4\pi}\left\{\beta_0\left(\frac{2}{n-4} + \gamma_E - \ln 4\pi\right) + C_A \frac{1}{3}\right\}\right], \quad (4)$$

and

$$\Gamma_{ij}(x) = \delta_{ij}\,\delta(1-x) + \frac{\alpha_s}{4\pi}\left[\frac{1}{2} P_{ij}(x)\left(\frac{2}{n-4} + \gamma_E - \ln 4\pi\right) + Z_{ij}(x)\right], \quad (5)$$

with [7][1]

$$Z = \begin{pmatrix} Z_{qq} & Z_{qg} \\ Z_{gq} & Z_{gg} \end{pmatrix} = \begin{pmatrix} C_F\left[-2 + 2x + \delta(1-x)\right] & T_f\left[-4x + 4x^2\right] \\ C_F\left[-2x\right] & C_A\left[\delta(1-x)/3\right] \end{pmatrix}. \quad (6)$$

For $SU(N)$ we have $C_A = N$, $C_F = (N^2-1)/2N$ and $T_f = 1/2$. In QCD we have $N = 3$.

In this paper we shall concentrate on the radiative graphs and reserve some comments on the loop graphs for the end. This implies that we will limit our discussions to the regular parts of the kernels Γ_{ij} and postpone the treatment of the singular parts represented by the $\delta(1-x)$ terms to later on. With the above finite coupling constant renormalization and finite mass factorization the jet cross sections in hadron–hadron collisions [1, 2] could be brought into agreement with each other. However for processes which have an additional mass scale this was not successful [8]. Here we have additional terms which however only multiply the mass dependent parts of the Born cross sections. Therefore these are not finite mass factorizations because they would involve the whole Born cross sections. These additional terms are given by

$$K_i = C_i \frac{\alpha_s}{4\pi}\left[-4\frac{1-x}{x}\right], \quad C_q = C_F, \quad C_g = C_A, \quad (7)$$

and are universal. Careful examination shows that they only occur for unpolarized processes which have a gluon exchange in a t-channel or a u-channel graph; see for instance the diagrams in Fig. 1. The terms are of the type m^2/t^2 or m^2/u^2 where m is the additional mass scale. For polarized processes this phenomenon does not occur because these terms do not exist. This can be inferred from (7) because of the term $1/x$ which is characteristic for unpolarized processes which have a gluon exchange in the t-channel or u-channel. We now examine specific reactions.

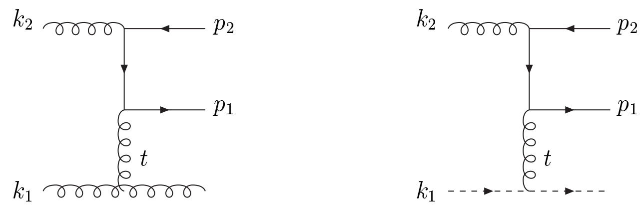

Fig. 1. t-channel graphs for the heavy flavor production processes $g + g \to Q + \bar{Q} + g$ and $g + q \to Q + \bar{Q} + q$

Let us start with heavy flavor production in hadron–hadron collisions. In [8] the cross sections were calculated in both regularization schemes. For the gg channel the Born cross section can be written as follows (see (2.5)–(2.11) in [8])

$$\begin{aligned}
&g(k_1) + g(k_2) \to Q(p_1) + \bar{Q}(p_2),\\
&s = (k_1 + k_2)^2, \quad t_1 = (k_2 - p_2)^2 - m^2,\\
&u_1 = (k_1 - p_2)^2 - m^2,\\
&s^2 \frac{d^2\sigma^{(0)}_{gg}}{dt_1 du_1} = s^2 \frac{d^2\sigma^{(0)}_{gg,O}}{dt_1 du_1} + s^2 \frac{d^2\sigma^{(0)}_{gg,K}}{dt_1 du_1},\\
&s^2 \frac{d^2\sigma^{(0)}_{gg,O}}{dt_1 du_1} = \pi\alpha_s^2 \frac{N}{2(N^2-1)}\left[\frac{t_1^2 + u_1^2}{s^2}\right] \times B_{\mathrm{QED}}\,\delta(s + t_1 + u_1),\\
&s^2 \frac{d^2\sigma^{(0)}_{gg,K}}{dt_1 du_1} = -\pi\alpha_s^2 \frac{1}{2N(N^2-1)} \times B_{\mathrm{QED}}\,\delta(s + t_1 + u_1),\\
&B_{\mathrm{QED}} = \frac{t_1}{u_1} + \frac{u_1}{t_1} + \frac{4m^2 s}{t_1 u_1}\left(1 - \frac{m^2 s}{t_1 u_1}\right). \quad (8)
\end{aligned}$$

We encounter for the first time differences between n-dimensional regularization and n-dimensional reduction in the (NLO) gg cross sections (see (6.16) and (6.17) in [8]). They are represented by the terms K_g convoluted with the

[1] In [7] the $-4x + 4x^2$ must be put in the upper right-hand corner

mass dependent parts of the Born cross sections indicated by the subscript m

$$s^2 \left.\frac{\mathrm{d}^2\sigma^{(1)}_{gg,i}}{\mathrm{d}t_1\mathrm{d}u_1}\right|_{\mathrm{reg}} = s^2 \left.\frac{\mathrm{d}^2\sigma^{(1)}_{gg,i}}{\mathrm{d}t_1\mathrm{d}u_1}\right|_{\mathrm{red}} + 2\,K_g \otimes s^2 \left.\frac{\mathrm{d}^2\sigma^{(0)}_{gg,i}}{\mathrm{d}t_1\mathrm{d}u_1}\right|_{m}, \quad i = O, K\,, \tag{9}$$

where the symbol $\otimes$ denotes the convolution integral. The gg Born cross sections can also be written in a different way, namely

$$s^2 \frac{\mathrm{d}^2\sigma^{(0)}_{gg,F}}{\mathrm{d}t_1\mathrm{d}u_1} = \pi\,\alpha_{\mathrm{s}}^2 \frac{C_F}{N^2-1} B_{\mathrm{QED}}\,\delta(s+t_1+u_1)\,,$$
$$s^2 \frac{\mathrm{d}^2\sigma^{(0)}_{gg,A}}{\mathrm{d}t_1\mathrm{d}u_1} = -\pi\,\alpha_{\mathrm{s}}^2 \frac{C_A}{N^2-1} \left(\frac{t_1 u_1}{s^2}\right) \times B_{\mathrm{QED}}\,\delta(s+t_1+u_1)\,. \tag{10}$$

Moreover we have the Born cross section for the $q\bar{q} \to Q\bar{Q}$ reaction

$$s^2 \frac{\mathrm{d}^2\sigma^{(0)}_{q\bar{q}}}{\mathrm{d}t_1\mathrm{d}u_1} = \pi\,\alpha_{\mathrm{s}}^2 \frac{C_F}{N} A_{\mathrm{QED}}\,\delta(s+t_1+u_1)\,,$$
$$A_{\mathrm{QED}} = \frac{t_1^2+u_1^2}{s^2} + \frac{2\,m^2}{s}\,. \tag{11}$$

In this way (4.23) in [9] can be written as

$$s^2 \left.\frac{\mathrm{d}^2\sigma^{(1)}_{g\bar{q},F}}{\mathrm{d}t_1\mathrm{d}u_1}\right|_{\mathrm{reg}} = s^2 \left.\frac{\mathrm{d}^2\sigma^{(1)}_{g\bar{q},F}}{\mathrm{d}t_1\mathrm{d}u_1}\right|_{\mathrm{red}} + Z_{gq} \otimes s^2 \frac{\mathrm{d}^2\sigma^{(0)}_{gg,F}}{\mathrm{d}t_1\mathrm{d}u_1} + Z_{qg} \otimes s^2 \frac{\mathrm{d}^2\sigma^{(0)}_{q\bar{q}}}{\mathrm{d}t_1\mathrm{d}u_1} + K_q \otimes s^2 \left.\frac{\mathrm{d}^2\sigma^{(0)}_{gg,F}}{\mathrm{d}t_1\mathrm{d}u_1}\right|_{m}, \tag{12}$$

and (4.24) becomes equal to

$$s^2 \left.\frac{\mathrm{d}^2\sigma^{(1)}_{g\bar{q},A}}{\mathrm{d}t_1\mathrm{d}u_1}\right|_{\mathrm{reg}} = s^2 \left.\frac{\mathrm{d}^2\sigma^{(1)}_{g\bar{q},A}}{\mathrm{d}t_1\mathrm{d}u_1}\right|_{\mathrm{red}} + Z_{gq} \otimes s^2 \frac{\mathrm{d}^2\sigma^{(0)}_{gg,A}}{\mathrm{d}t_1\mathrm{d}u_1} + K_q \otimes s^2 \left.\frac{\mathrm{d}^2\sigma^{(0)}_{gg,A}}{\mathrm{d}t_1\mathrm{d}u_1}\right|_{m}. \tag{13}$$

Both cross sections involve extra terms which are proportional to K_q convoluted with the mass dependent parts of the cross sections. Finally the $q\bar{q}$ cross section behaves in a normal way and (4.8) in [9] becomes

$$s^2 \frac{\mathrm{d}^2\sigma^{(1)}_{q\bar{q},F}}{\mathrm{d}t_1\mathrm{d}u_1}_{\mathrm{reg}} = s^2 \frac{\mathrm{d}^2\sigma^{(1)}_{q\bar{q},F}}{\mathrm{d}t_1\mathrm{d}u_1}_{\mathrm{red}} + 2\,Z_{qq} \otimes s^2 \frac{\mathrm{d}^2\sigma^{(0)}_{q\bar{q}}}{\mathrm{d}t_1\mathrm{d}u_1}\,. \tag{14}$$

The next process is electroproduction of heavy flavors. Here two mass scales are involved i.e. the heavy flavor mass m and the virtuality of the off-shell photon $Q^2 = -q^2$. The Born cross sections for the transverse (G) and longitudinal (L) parts are (see (2.14) and (2.15) in [10])

$$\gamma^*(q) + g(k_1) \to Q(p_1) + \bar{Q}(p_2)\,,$$
$$\begin{aligned}
s &= (q+k_1)^2 = s' + q^2\,,\\
t_1 &= (k_1-p_2)^2 - m^2\,,\\
u_1 &= (q-p_2)^2 - m^2 = u_1' + q^2\,,\\
s'^2 \frac{\mathrm{d}^2\sigma^{(0)}_{i,g}}{\mathrm{d}t_1\mathrm{d}u_1} &= \pi\, e_H^2\,\alpha\,\alpha_{\mathrm{s}}\,a_i\,B_{i,\mathrm{QED}}\,\delta(s'+t_1+u_1)\,,\\
i &= G, L\,, \quad a_G = 1\,, \quad a_L = 2\,,\\
B_{G,\mathrm{QED}} &= \frac{t_1}{u_1} + \frac{u_1}{t_1} + \frac{4\,m^2\,s'}{t_1\,u_1}\left(1 - \frac{m^2\,s'}{t_1\,u_1}\right)\\
&\quad + \frac{2\,s'\,q^2}{t_1\,u_1} + \frac{2\,q^4}{t_1\,u_1}\\
&\quad + \frac{2\,m^2\,q^2}{t_1\,u_1}\left(2 - \frac{s'^2}{t_1\,u_1}\right),\\
B_{L,\mathrm{QED}} &= -\frac{4\,q^2}{s'}\left(1 - \frac{q^2}{s'} - \frac{m^2\,s'}{t_1\,u_1}\right),\\
q^2 &= -Q^2\,.
\end{aligned} \tag{15}$$

The differences between n-dimensional regularization and n-dimensional reduction are visible in the NLO off-shell photon–gluon fusion processes. Equations (4.7) and (4.8) in [10] are equal to

$$s^2 \left.\frac{\mathrm{d}^2\sigma^{(1)}_{i,g}}{\mathrm{d}t_1\mathrm{d}u_1}\right|_{\mathrm{reg}} = s^2 \left.\frac{\mathrm{d}^2\sigma^{(1)}_{i,g}}{\mathrm{d}t_1\mathrm{d}u_1}\right|_{\mathrm{red}} + K_g \otimes s^2 \left.\frac{\mathrm{d}^2\sigma^{(0)}_{i,g}}{\mathrm{d}t_1\mathrm{d}u_1}\right|_{m,Q}, \quad i = G, L\,, \tag{16}$$

where the second terms of the above equation contains all pieces proportional to m^2 and q^2 in the Born cross sections in (15). For the Bethe–Heitler process (A_1) in off-shell photon–quark scattering we see the same phenomenon. Equation (4.11) in [10] becomes

$$s^2 \left.\frac{\mathrm{d}^2\sigma^{(1)}_{i,q,A_1}}{\mathrm{d}t_1\mathrm{d}u_1}\right|_{\mathrm{reg}} = s^2 \left.\frac{\mathrm{d}^2\sigma^{(1)}_{i,q,A_1}}{\mathrm{d}t_1\mathrm{d}u_1}\right|_{\mathrm{red}} + Z_{gq} \otimes s^2 \frac{\mathrm{d}^2\sigma^{(0)}_{i,g}}{\mathrm{d}t_1\mathrm{d}u_1} + K_q \otimes s^2 \left.\frac{\mathrm{d}^2\sigma^{(0)}_{i,g}}{\mathrm{d}t_1\mathrm{d}u_1}\right|_{m,Q}, \quad i = G, L\,. \tag{17}$$

For $i = G$ we get the same in the case of on-shell photon–hadron production ($q^2 = 0$) [11] except that now also the Compton process (A_2) gets a collinear divergence. The difference between both regularizations in (4.17) of [11] becomes

$$s^2 \left.\frac{\mathrm{d}^2\sigma^{(1)}_{\gamma q,A_2}}{\mathrm{d}t_1\mathrm{d}u_1}\right|_{\mathrm{reg}} = s^2 \left.\frac{\mathrm{d}^2\sigma^{(1)}_{\gamma q,A_2}}{\mathrm{d}t_1\mathrm{d}u_1}\right|_{\mathrm{red}} + Z_{qg} \otimes s^2 \frac{\mathrm{d}^2\sigma^{(0)}_{q\bar{q}}}{\mathrm{d}t_1\mathrm{d}u_1}\,, \tag{18}$$

which is of the usual form.

Finally we turn our attention to the Drell–Yan process. We look at the differential distributions of the vector boson with momentum q. The cross sections have been calculated in n-dimensional regularization in [12, 13]. We have also calculated them using n-dimensional reduction. The Born processes and Born cross sections are given by

$$
\begin{aligned}
&q(p_1) + \bar{q}(p_2) \to \gamma^*(q) + g(k)\,,\\
&q(p_1) + g(p_2) \to \gamma^*(q) + q(k)\,,\\
&s = (p_1+p_2)^2\,, \quad t = (p_1-q)^2\,,\\
&u = (p_2-q)^2\,, \quad q^2 = Q^2\,,\\
&s^2\,\frac{\mathrm{d}^2 W^{(0)}_{q\bar{q}}}{\mathrm{d}t\mathrm{d}u} = \alpha_\mathrm{s}\,\frac{C_F}{N}\left[\frac{4\,s\,Q^2 + 2\,t^2 + 2\,u^2}{t\,u}\right]\\
&\qquad \times\delta(s+t+u-Q^2)\,,\\
&s^2\,\frac{\mathrm{d}^2 W^{(0)}_{qg}}{\mathrm{d}t\mathrm{d}u}\\
&\quad = \alpha_\mathrm{s}\,\frac{T_f}{N}\left[-\frac{4\,Q^2\,(Q^2-s-u) + 2\,s^2 + 2\,u^2}{s\,u}\right]\\
&\qquad \times\delta(s+t+u-Q^2)\,. \qquad (19)
\end{aligned}
$$

The NLO $q\bar{q}$ process involves no problem. We find

$$
s^2\,\frac{\mathrm{d}^2 W^{(1)}_{q\bar{q}}}{\mathrm{d}t\mathrm{d}u}\bigg|_{\mathrm{reg}} = s^2\,\frac{\mathrm{d}^2 W^{(1)}_{q\bar{q}}}{\mathrm{d}t\mathrm{d}u}\bigg|_{\mathrm{red}} + 2\,Z_{qq}\otimes s^2\,\frac{\mathrm{d}^2 W^{(0)}_{q\bar{q}}}{\mathrm{d}t\mathrm{d}u}\,. \qquad (20)
$$

However for the NLO qg and qq subprocesses differences between n-dimensional regularization and n-dimensional reduction appear again and equal the mass (here Q^2) dependent part of the Born cross sections convoluted with either K_g (qg) or K_q (qq).

$$
\begin{aligned}
s^2\,\frac{\mathrm{d}^2 W^{(1)}_{qg}}{\mathrm{d}t\mathrm{d}u}\bigg|_{\mathrm{reg}} &= s^2\,\frac{\mathrm{d}^2 W^{(1)}_{qg}}{\mathrm{d}t\mathrm{d}u}\bigg|_{\mathrm{red}} + Z_{qq}\otimes s^2\,\frac{\mathrm{d}^2 W^{(0)}_{qg}}{\mathrm{d}t\mathrm{d}u} \qquad (21)\\
&\quad + Z_{qg}\otimes s^2\,\frac{\mathrm{d}^2 W^{(0)}_{q\bar{q}}}{\mathrm{d}t\mathrm{d}u} + K_g\otimes\, s^2\,\frac{\mathrm{d}^2 W^{(0)}_{qg}}{\mathrm{d}t\mathrm{d}u}\bigg|_Q\,,
\end{aligned}
$$

$$
\begin{aligned}
s^2\,\frac{\mathrm{d}^2 W^{(1)}_{qq}}{\mathrm{d}t\mathrm{d}u}\bigg|_{\mathrm{reg}} &= s^2\,\frac{\mathrm{d}^2 W^{(1)}_{qq}}{\mathrm{d}t\mathrm{d}u}\bigg|_{\mathrm{red}} + 2\,Z_{gq}\otimes s^2\,\frac{\mathrm{d}^2 W^{(0)}_{qg}}{\mathrm{d}t\mathrm{d}u}\\
&\quad + 2\,K_q\otimes\, s^2\,\frac{\mathrm{d}^2 W^{(0)}_{qg}}{\mathrm{d}t\mathrm{d}u}\bigg|_Q\,. \qquad (22)
\end{aligned}
$$

Finally the NLO gg subprocess behaves like the $q\bar{q}$ subprocess and does need this extra term, so

$$
s^2\,\frac{\mathrm{d}^2 W^{(1)}_{gg}}{\mathrm{d}t\mathrm{d}u}\bigg|_{\mathrm{reg}} = s^2\,\frac{\mathrm{d}^2 W^{(1)}_{gg}}{\mathrm{d}t\mathrm{d}u}\bigg|_{\mathrm{red}} + 2\,Z_{qg}\otimes s^2\,\frac{\mathrm{d}^2 W^{(0)}_{qg}}{\mathrm{d}t\mathrm{d}u}\,. \qquad (23)
$$

In all the above reactions we observe that when the mass dependent part of the cross section appears convoluted with K_k ($k = q, g$) we also encounter the exchange of a gluon in t- or u-channel graphs.

To decide which regularization prescription is correct we try out another regularization technique. Here we choose the off-shell technique [14–16] which is defined so that all external particles are taken off-shell $p_i^2 < 0$. The intrinsic particle masses are equal to zero and the collinear divergences appear as $\ln(-Q^2/p^2)$. The kernels Γ_{ij} become equal to the operator matrix elements where the external legs are put off-shell. In this case the regular part of Γ_{ij} in the $\overline{\mathrm{MS}}$ scheme becomes

$$
\begin{aligned}
\Gamma_{ij}(x) &= \delta_{ij}\,\delta(1-x) \qquad (24)\\
&\quad + \frac{\alpha_\mathrm{s}}{4\pi}\left[\frac{1}{2}\,P_{ij}(x)\,\ln\left(\frac{\mu^2}{-x\,(1-x)\,p^2}\right) + Z_{ij}(x)\right]\,,
\end{aligned}
$$

with the finite renormalization Z equal to

$$
Z(x) = \begin{pmatrix} C_F\,[-4+2\,x] & T_f\,[-2-4\,x\,(1-x)] \\ C_F\,[(-4+2\,x-2\,x^2)/x] & C_A\,[(5\,x-4)/x] \end{pmatrix}. \qquad (25)
$$

We omit the $\delta(1-x)$ terms in Z_{ij} because they concern the soft-plus-virtual gluon contributions. These terms are very complicated in the off-shell approach [17]. Substituting Γ_{ij} in the above equations we observe that $K_i = 0$, in other words we get the same as n-dimensional regularization. Apparently the $n-4$ terms appearing in the numerator by use of n-dimensional regularization, which are multiplied by pole terms $1/(n-4)$, mimic analogous terms in the numerator which are proportional to p^2 in the case of off-shell regularization and are multiplied by $1/p^2$. The latter terms arise in those parts of the cross sections which are proportional to p^2/t^2 or p^2/u^2. Therefore one cannot omit these terms. In n-dimensional reduction the $n-4$ terms are not present and $p^2 = 0$ is put at the beginning. This leads us to the conclusion that for QCD processes with an additional mass scale n-dimensional reduction is wrong unless one wants to add an additional mass factorization which however is not proportional to the whole Born cross section.

Finally we have also studied the soft-plus-virtual gluon contributions in the n-dimensional regularization and n-dimensional reduction methods. Since in the loop graphs UV divergences also appear we only get consistency between both regularizations if we choose an $\mathcal{N} = 1$ supersymmetry where the quarks are now Majorana fermions in the adjoint representation. Therefore $C_A = C_F = n_f = N$ for $SU(N)$. For the Drell–Yan $q\bar{q}$ process we get consistency provided we implement the finite coupling constant renormalization in (4) and the finite mass factorization in (6) for the $\delta(1-x)$ terms. However for the qg process we get an inconsistency unless we put a factor of $3/2$ instead of a one in the coefficient of the term containing the $\delta(1-x)$ function in Z_{qq} of (6). This is in disagreement with what we found for the Drell–Yan $q\bar{q}$ process and with the jet processes in [1, 2][2]. In conclusion we find a disagreement in the radiative part of the NLO cross sections between n-dimensional regularization and n-dimensional reduction for processes which involve an additional mass scale. However the off-shell regularization method indicates that n-dimensional regularization yields the correct answer.

[2] After the submission of this paper our attention was called to a similar phenomenon discovered in [18].

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